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Flow-induced flutter instability of cantilever carbon nanotubes

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Abstract

Carbon nanotubes are finding significant application to nanofluidic devices. This work studies the influence of internal moving fluid on free vibration and flow-induced flutter instability of cantilever carbon nanotubes based on a continuum elastic model. Since the flow-induced vibration of cantilever pipes is non-conservative in nature, cantilever carbon nanotubes conveying fluid are damped with decaying amplitude for flow velocity below a certain critical value. Beyond this critical flow velocity, flutter instability occurs and vibration becomes amplified with growing amplitude. Our results indicate that internal moving fluid substantially affects vibrational frequencies and the decaying rate of amplitude especially for longer cantilever carbon nanotubes of larger innermost radius at higher flow velocity, and the critical flow velocity for flutter instability in some cases may fall within the practical range. On the other hand, a moderately stiff surrounding elastic medium (such as polymers) can significantly suppress the effect of internal moving fluid on vibrational frequencies and suppress or eliminate flutter instability within the practical range of flow velocity.

Keywords: Vibration; Flutter; Carbon nanotubes; Nanopipe

1. Introduction

Because of perfect hollow cylindrical geometry and superior mechanical strength, carbon nanotubes (CNTs) hold substantial promise as nanocontainers for gas storage, and nanopipes conveying fluid (such as gases or water) (Evans and Bowman, 1996; Gadd and Blackford, 1997; Liu et al., 1998; Che et al., 1998; Hummer et al., 2001; Karlsson, 2001; Gao and Bando, 2002). Fluid flow inside CNTs raises a significant and challenging research topic. On the other hand, the influence of internal moving fluid on overall mechanical behavior of CNTs is another topic of major concern.

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Fluid mechanics within CNTs aims to study how the wall–fluid interaction and the viscosity of fluid affect velocity distribution inside CNTs, and how the velocity distribution (and the mean flow velocity) depends on the applied pressure gradient in a non-classic way (Tuzun et al., 1996; Mao and Sinnott, 2000; Gogotsi, 2001; Megaridis et al., 2002; Galanov et al., 2002; Sokhan et al., 2002; Skoulidas et al., 2002; Supple and Quirke, 2003). Since our goal is not to study fluid mechanics inside CNTs, we shall assume, instead, that a uniform steady-state flow is achieved throughout a straight CNT, with a constant and uniform (mean) flow velocity defined by the flow flux divided by the innermost cross-sectional area. Here, it is stated that the uniformity of flow velocity throughout the entire CNT is a simple consequence of the uniform cross-section if the fluid is assumed to be incompressible. Thus, the role of the internal moving fluid is characterized by two parameters, the (mean) flow velocity U and the mass density of fluid M (per unit axial length). The wall–fluid interaction and the viscosity of fluid, which substantially affect the velocity distribution and the mean flow velocity U, will not explicitly appear in the present study. In other words, the role of the wall–fluid interaction and the viscosity of fluid is included only implicitly through their influence on the mean flow velocity. Based on these ideas, we have recently studied the effects of internal moving fluid on free vibration and structural instability of CNTs pinned or clamped at both ends (Yoon et al., 2005). In that case, internal flow-induced vibration of CNTs is conservative in nature and characterized by periodic vibration with constant amplitude, and the lowest frequency reduces to zero when a critical flow velocity is reached. This leads to "divergence instability" of supported CNTs, similar to static buckling of compressed elastic column. Our results (Yoon et al., 2005) show that internal moving fluid substantially affects vibrational frequencies especially for longer CNTs of larger innermost radius at higher flow velocity, and the critical flow velocity for "divergence instability" of CNTs in some cases may fall within the range of practical significance.

Many proposed applications of CNTs as nanopipes are likely involved with cantilever CNTs which are clamped at one end but free at the other end. It is known that flow-induced vibration of cantilever pipes is non-conservative in nature and characterized by decaying or growing amplitude (Chen, 1987; Paidoussis and Li, 1993; Paidoussis, 1998). When the flow velocity is sufficiently low, vibration of cantilever pipes fades off with time. On the other hand, vibration amplitude will grow with time after a critical flow velocity is reached. This phenomenon is called "flutter" which has been studied extensively within the framework of aeroelasticity (Fung, 1993). Motivated by the idea that vibration and flutter instability of cantilever CNTs conveying fluid are likely of both theoretical and practical interest, the present paper studies flow induced free vibration and flutter instability of cantilever CNTs. Here, the structural behavior of CNTs is described by the classic Euler-beam model (Ru, 2004; Yoon et al., 2005), and the role of internal moving fluid is characterized by two parameters, the mean flow velocity U and the mass density of fluid M (per unit axial length), as shown in Fig. 1. As will be shown below, internal moving fluid has a substantial effect on vibrational frequencies and the decaying rate of amplitude especially for longer CNTs of larger innermost radius at higher flow velocity, and the critical flow velocity for flutter instability could fall within the range of practical significance at least in some extreme cases. On the other hand, a moderately stiff surrounding elastic medium (such as polymer) can suppress or eliminate the influence of internal moving fluid on both vibrational frequencies and flutter instability of CNTs conveying fluid.

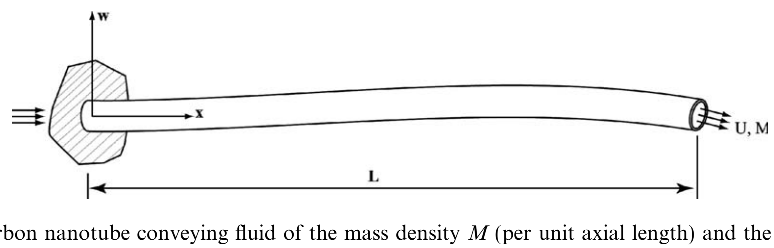

Fig. 1. Cantilever carbon nanotube conveying fluid of the mass density M (per unit axial length) and the mean flow velocity U.

2. The model for cantilever CNTs conveying fluid

As shown in Fig. 1, a CNT conveying fluid will be described by a cantilever elastic hollow tube. As addressed in Ru (2004), continuum elastic-beam models have been effectively used to study static and dynamic structural behavior of CNTs, such as static deflection (Wong et al., 1997), column buckling (Garg et al., 1998), resonant frequencies and modes (Treacy et al., 1996; Poncharal et al., 1999), and sound wave propagation (Popov et al., 2000). These studies showed that the classic Euler-elastic beam offers a simple and reliable model for overall mechanical deformation of CNTs provided the characteristic wave-length is much larger than the diameter of CNTs. For example, buckling force predicted by the Euler-beam model is in good agreement with experimental data (Garg et al., 1998), resonant frequencies and vibrational modes of CNTs given by the cantilever beam model agree well with experiments (Treacy et al., 1996; Poncharal et al., 1999), and sound velocity predicted by the Euler-beam model agrees well with the data obtained by other methods (Popov et al., 2000). In particular, non-coaxial resonance of multiwall carbon nanotubes (MWNTs) first predicted by a simple multiple-beam model (Yoon et al., 2002) is found to well agree with more recent atomistic simulations (Zhao et al., 2003; Li and Chou, 2004). Since elastic beam models enjoy simple mathematical formulas, they have the potential to identify the key parameters affecting basic mechanical behavior of CNTs (and thus rule out other less important parameters), predict new physical phenomena, and stimulate and guide further experiments and molecular dynamics simulations.

Here, as usual, we shall neglect gravity effect and assume that the constraint for axial displacement of the cantilever CNT is absent or negligible. Thus, vibration and flutter instability of a cantilever CNT conveying fluid can be described by the model (Chen, 1987; Paidoussis and Li, 1993; Paidoussis, 1998; Yoon et al., 2005)

$$EI\frac{\partial^4 w}{\partial x^4} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x\,\partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} + Kw = 0 \tag{1}$$

where x is the axial coordinate, t is time, $w(x,t)$ is the deflection of the CNT, E and I are Young's modulus and the moment of inertia of the cross-section of the CNT, m is the mass of CNT per unit axial length (which is equal to the cross-sectional area of CNT multiplied by the mass density of CNTs), K is the Winkler constant of the surrounding elastic medium described as a Winkler-like elastic foundation (Paidoussis and Li, 1993; Paidoussis, 1998; Yoon et al., 2005). In addition, the externally imposed tension and pressurization are absent.

It is stressed that the role of internal moving fluid is characterized by two parameters of the fluid, its mass density M (per unit axial length) and the mean flow velocity U (defined by the flow flux divided by the area of the innermost cross-section of CNT). The wall–fluid interaction and the viscosity of fluid inside CNTs do affect vibration and instability of CNT, but only through affecting the velocity distribution and the mean flow velocity. Hence, the effect of the wall–fluid interaction and the viscosity of fluid is implicitly included in the velocity distribution and the mean flow velocity, and will not explicitly appear in the governing Eq. (1). Therefore, the present work focuses on the effects of internal moving fluid on vibration and flutter instability, without concerning how the wall–fluid interaction and the viscosity of fluid affect the mean flow velocity U or what applied pressure gradient is required to achieve such a flow inside CNTs.

To highlight the non-conservative nature of flow-induced flutter instability of cantilever CNTs, it is stated that, as explained by Benjamin (1961) and Paidoussis and Li (1993), the work (ΔW) done by the fluid forces to the elastic tube over a cycle of oscillation of period T is

$$\Delta W = -MU\int_0^T \left[\left(\frac{\partial w}{\partial t}\right)^2 + U\left(\frac{\partial w}{\partial t}\right)\left(\frac{\partial w}{\partial x}\right)\right]_L \mathrm{d}t - \frac{1}{2}M\int_0^T \left[\left(\frac{\partial w}{\partial t}\right)^2 + U^2\left(\frac{\partial w}{\partial x}\right)^2\right]_0^T \mathrm{d}x \tag{2}$$

where $(\partial w/\partial t)_L$ and $(\partial w/\partial x)_L$ are the lateral velocity and the slope at the end $x = L$, respectively. For supported CNTs, because $(\partial w/\partial t)_L$ is identically zero, the first integral on RHS is zero. In addition, because the vibration is strictly periodic, the second term on RHS also vanishes. Hence, $\Delta W = 0$ and vibration of supported CNTs is conservative. For a cantilever CNT, however, because the deflection and slope of the free end are not identically zero and the amplitude at $t = T$ is not exactly the same as its value at $t = 0$, none of the two integrals on RHS is identically zero. When U is sufficiently small, it turns out that the first term within the first square brackets is dominant over all other terms, it follows that $\Delta W < 0$, and thus the amplitude decays with time and the cantilever CNT is damped. However, this ceases to be true for sufficiently high flow velocity U which could lead to $\Delta W > 0$. Thus cantilever CNTs could gain energy from the flow, and vibration would be amplified for sufficiently high flow velocity. In other words, for sufficiently high flow velocity, cantilever CNTs could lose stability by flutter. We believe that flutter instability of cantilever CNTs has significant consequences to the design of CNTs as nanopipes conveying fluid.

3. Results and discussions

For free vibration of a cantilever CNT shown in Fig. 1, boundary conditions are

$$w(0,t) = \frac{\partial w}{\partial x}(0,t) = \frac{\partial^2 w}{\partial x^2}(L,t) = \frac{\partial^3 w}{\partial x^3}(L,t) = 0 \tag{3}$$

Consider solutions of the form

$$w(x,t) = \mathrm{Re}[C\mathrm{e}^{\mathrm{i}\alpha x}\mathrm{e}^{\mathrm{i}\omega t}] \tag{4}$$

where C is a constant, and ω is the complex circular frequency. Substitution of (4) into Eq. (1) gives

$$EI\alpha^4 - MU^2\alpha^2 - 2MU\omega\alpha - (M+m)\omega^2 + K = 0 \tag{5}$$

which determines four complex roots α_n $(n = 1,2,3,4)$ as a function of ω. The complete solution of Eq. (1) is thus

$$w(x,t) = \mathrm{Re}\left[\sum_{n=1}^{4} C_n \mathrm{e}^{\mathrm{i}\alpha_n x}\mathrm{e}^{\mathrm{i}\omega t}\right] \tag{6}$$

where the constants C_n $(n = 1,2,3,4)$ should be determined by the boundary condition (3). Thus, substituting (6) into (3) yields

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \alpha_1^2\mathrm{e}^{\mathrm{i}\alpha_1} & \alpha_2^2\mathrm{e}^{\mathrm{i}\alpha_2} & \alpha_3^2\mathrm{e}^{\mathrm{i}\alpha_3} & \alpha_4^2\mathrm{e}^{\mathrm{i}\alpha_4} \\ \alpha_1^3\mathrm{e}^{\mathrm{i}\alpha_1} & \alpha_2^3\mathrm{e}^{\mathrm{i}\alpha_2} & \alpha_3^3\mathrm{e}^{\mathrm{i}\alpha_3} & \alpha_4^3\mathrm{e}^{\mathrm{i}\alpha_4} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = 0 \tag{7}$$

The condition for existence of a non-trivial solution gives the characteristic equation in ω which determines the eigenvalues and the associated vibrational modes. For very small U, it turns out that the imaginary parts of all eigenvalues, which represent the decaying rate of amplitude, are non-negative and hence the vibration amplitude decays with time. As U increases, the imaginary parts of ω vary and at least one of them will reduce to zero at a certain critical flow velocity $U = U_c$ beyond which the imaginary part of ω changes sign from positive to negative and the amplitude will grow with time. This indicates the onset of flutter instability. In this paper, we shall confine ourselves to the effect of internal moving fluid on the first three vibrational modes of the CNT described by (1). Here, as usual, the first, second, and third vibrational modes are defined by the three lowest vibrational frequencies at $U = 0$. For $U > 0$, however, all frequencies vary with U. Hence, it is possible that the frequency of a higher mode may be even lower than the frequency of a lower mode for sufficiently high flow velocity U. For example, the frequency of mode 2 may be even lower than the frequency of mode 1 for sufficiently high U.

The examples of CNTs considered here are: (I) MWNT with the outermost radius $R = 40$ nm and thickness $h = 20$ nm, (II) MWNT with $R = 50$ nm and $h = 10$ nm (Galanov et al., 2002), where the thickness h is defined as the difference between the outermost and the innermost radii. Here, two different aspect ratios of CNTs, $L/2R = 10$ or 50, are considered. On the other hand, to be specific, water has been considered as the fluid inside CNTs. The mass density of CNTs is 2.3 g/cm^3 with Young's modulus E of 1 TPa, and the mass density of water is 1 g/cm^3. In addition, to study the effect of a surrounding elastic medium modeled as a Winkler-like elastic foundation, the Winkler-constant K is considered to have a value ranging from 1 KPa or 1 MPa (for soft materials such as bio-tissue, Shull, 2002), to 1 GPa (for moderately stiff materials like polymers). Finally, the available data in the literature for the flow velocity inside CNTs range from 400 m/s (Supple and Quirke, 2003) to 2000 m/s (Tuzun et al., 1996), or even up to 50 000 m/s (Mao and Sinnott, 2000). Therefore, to cover a wide range of flow velocity inside CNTs, we shall consider the flow velocity U up to 10 000 m/s, in spite of the fact that the available data for the flow velocity of water in CNTs (of very small innermost diameter) are much lower than this value.

In what follows, vibrational frequency $f = \mathrm{Re}(\omega/(2\pi))$ and the decaying rate $\mathrm{Im}(\omega/(2\pi))$ of cantilever CNTs conveying fluid are calculated and shown in Figs. 2–13 for CNTs with or without being embedded in a surrounding elastic medium characterized by the Winkler constant K. The main results are summarized as follows.

(1) The results of Figs. 2–13 indicate that internal moving fluid has a substantial effect on vibrational frequencies ($=\mathrm{Re}(\omega/(2\pi))$) and the decaying rate ($=\mathrm{Im}(\omega/(2\pi))$) of cantilever CNTs conveying fluid. This effect is more significant at higher flow velocity even for CNTs of smaller aspect ratio ($L/2R = 10$), and for CNTs of larger aspect ratio (such as $L/2R = 50$, as shown in Figs. 5–7, and 11–13) even at lower flow velocity, but is less significant for CNTs of smaller aspect ratio (such as $L/2R = 10$, as shown in Figs. 2–4 and 8–10) at lower flow velocity.

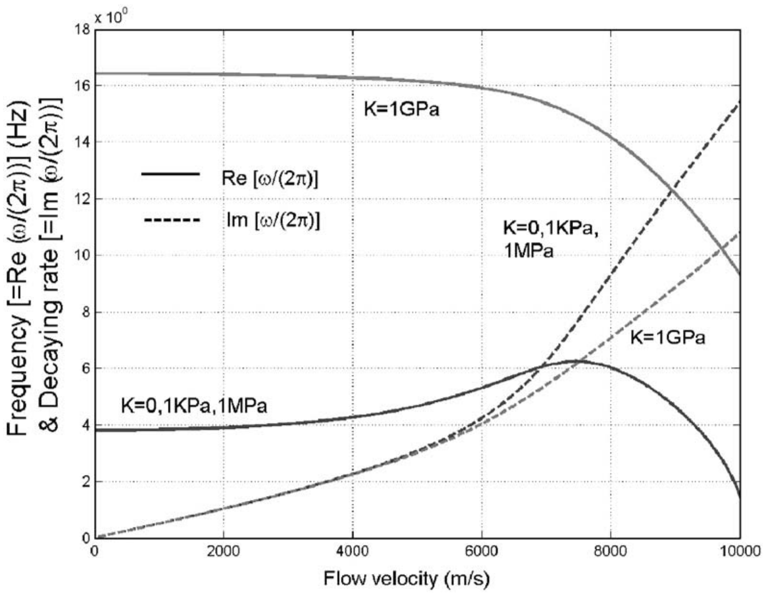

Fig. 2. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case I, $L/2R = 10$, mode 1).

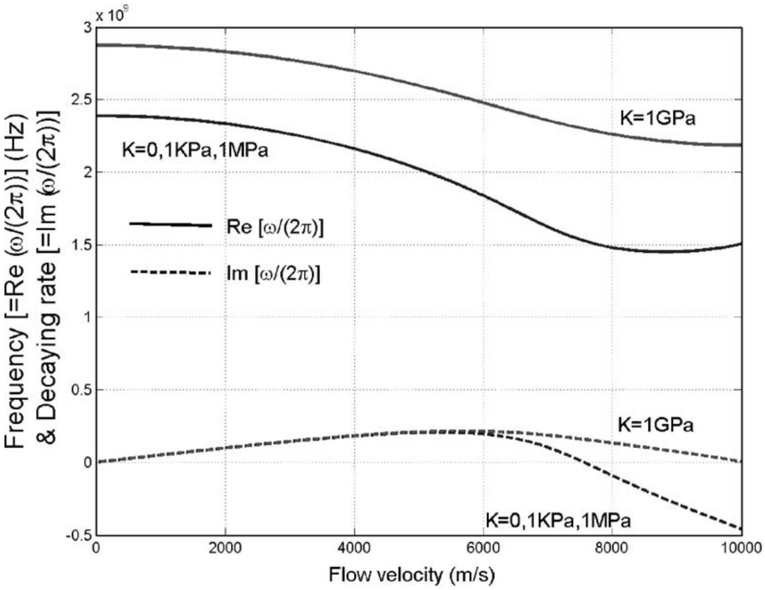

Fig. 3. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case I, $L/2R = 10$, mode 2).

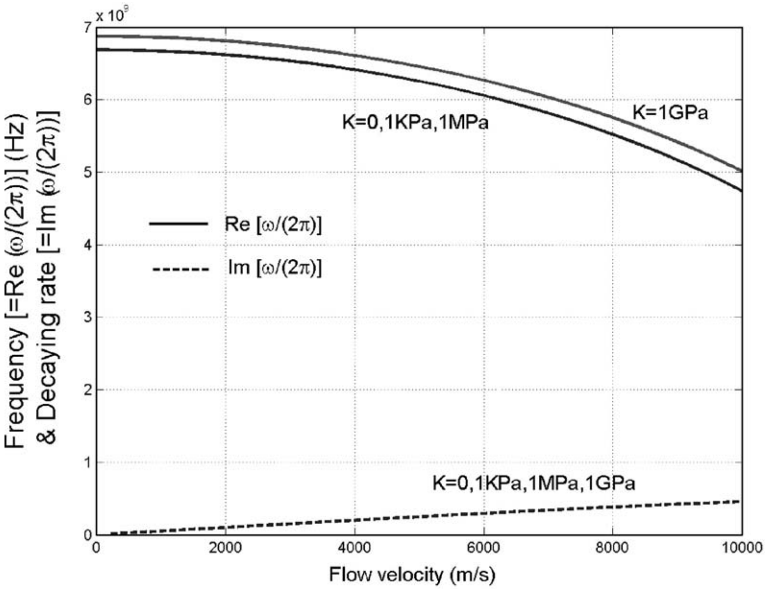

Fig. 4. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case I, $L/2R = 10$, mode 3).

(2) Vibrational frequencies ($=\mathrm{Re}(\omega/(2\pi))$) of CNTs conveying fluid vary with increasing flow velocity. For example, in Case I with $L/2R = 50$ and $K = 0$ (Fig. 5), vibrational frequency of mode 1 increases slowly up to $U = 1500$ m/s, then begins to decrease, and finally reduces to zero around $U = 2000$ m/s and remains zero until $U = 2700$ m/s. So, between $U = 2000$ m/s and 2700 m/s, the frequency of the first mode is iden-

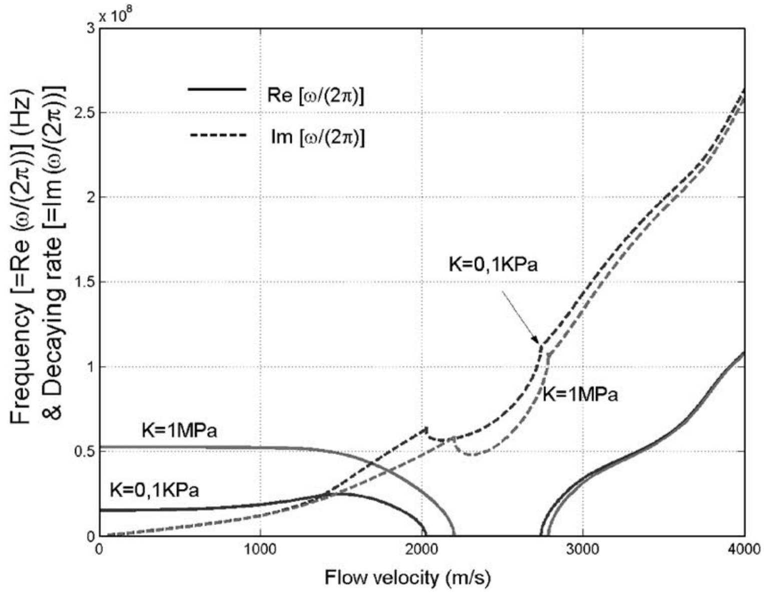

Fig. 5. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case I, $L/2R = 50$, mode 1).

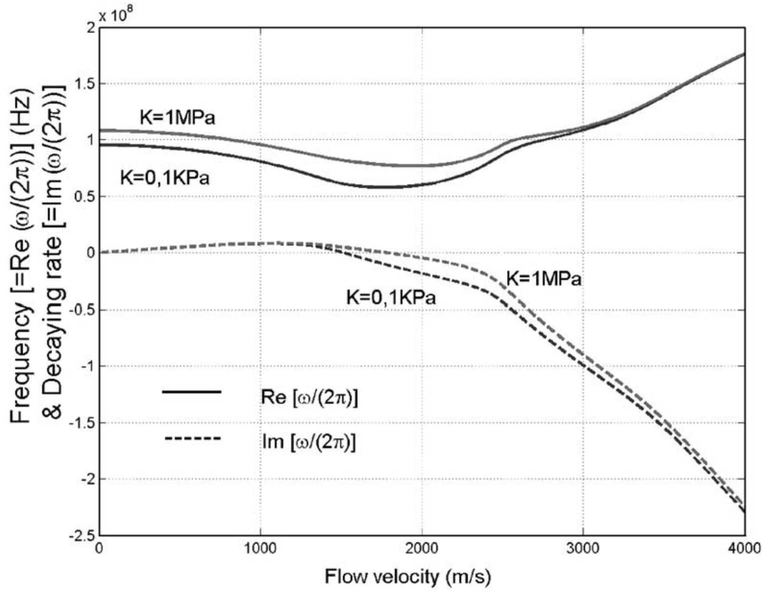

Fig. 6. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case I, $L/2R = 50$, mode 2).

tically zero (Fung, 1993), which means that the amplitude of deflection of the cantilever CNT decays monotonically with time without backward and forward oscillation. In addition, for Case II with $L/2R = 10$ and $K = 0$, it is seen from Fig. 8 that the frequency of mode 1 reduces to zero around $U = 2800$ m/s and remains

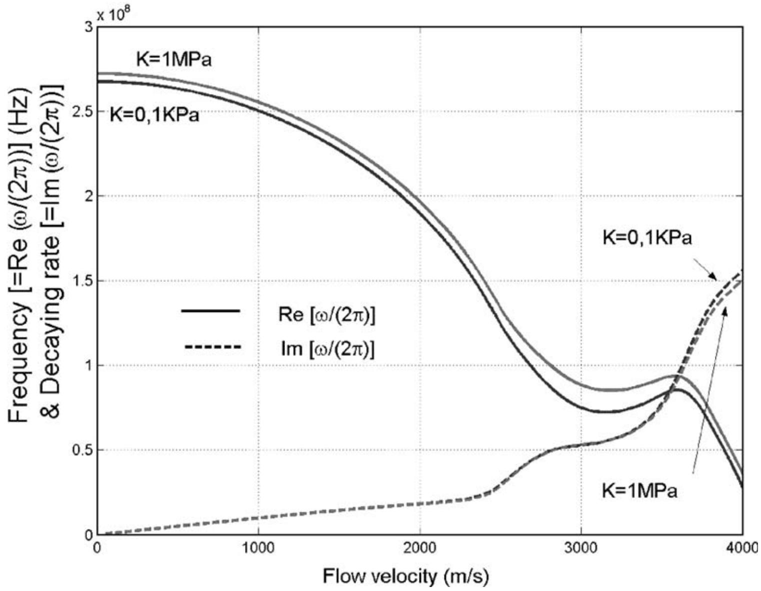

Fig. 7. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case I, $L/2R = 50$, mode 3).

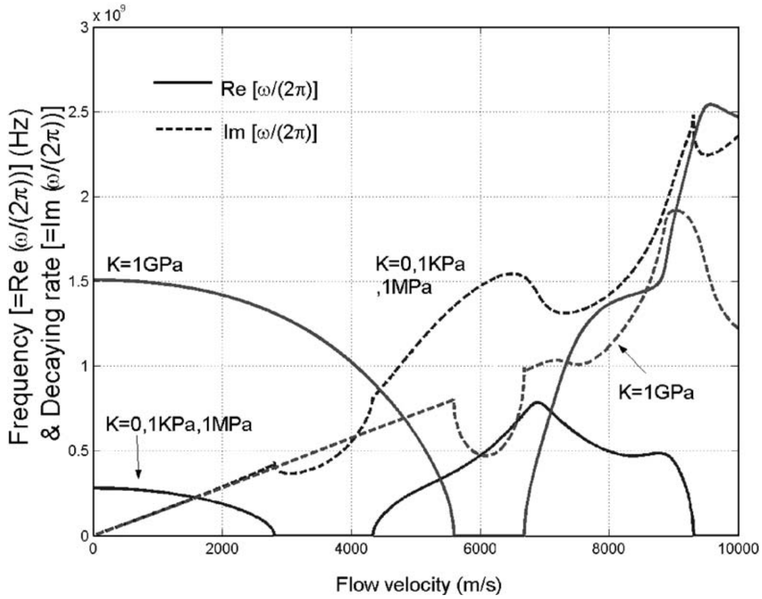

Fig. 8. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case II, $L/2R = 10$, mode 1).

zero until $U = 4300$ m/s. These phenomena qualitatively agree with the general results of cantilever elastic tubes conveying fluid (Paidoussis, 1998).

(3) For lower flow velocity U, internal flow causes damping to cantilever CNTs in all modes and the vibration amplitude of CNTs decays with time. For example, for $L/2R = 10$ and 50, the cantilever CNTs

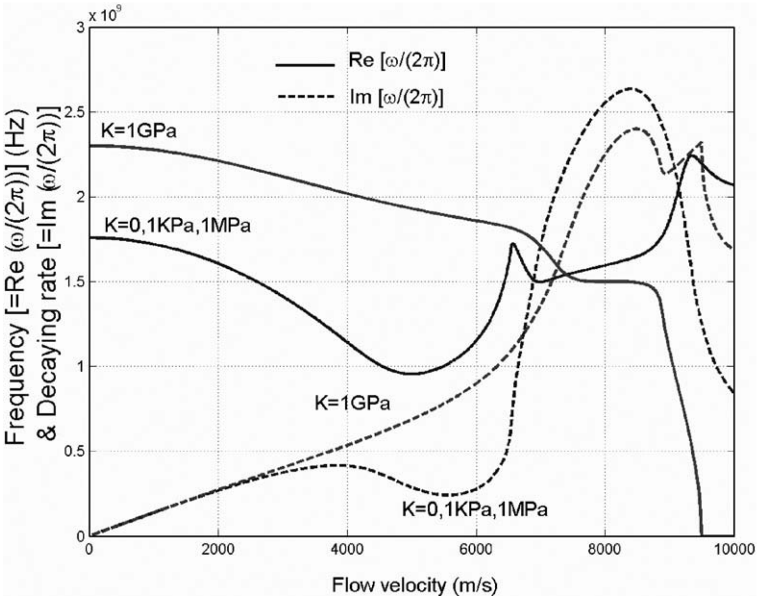

Fig. 9. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case II, $L/2R = 10$, mode 2).

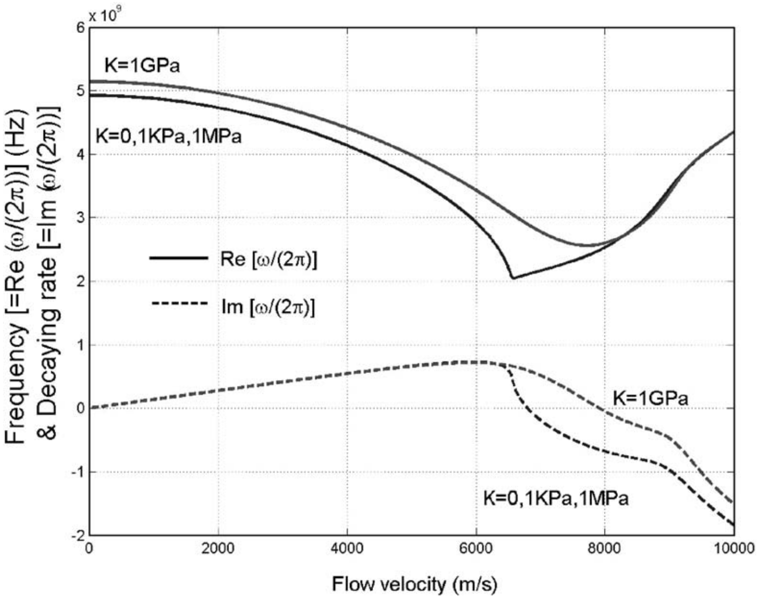

Fig. 10. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case II, $L/2R = 10$, mode 3).

in all cases discussed here are damped for flow velocity below 6000 m/s, and 1200 m/s, respectively. This phenomenon is common for the examples I, II, with or without a surrounding elastic medium, as shown in Figs. 2–13.

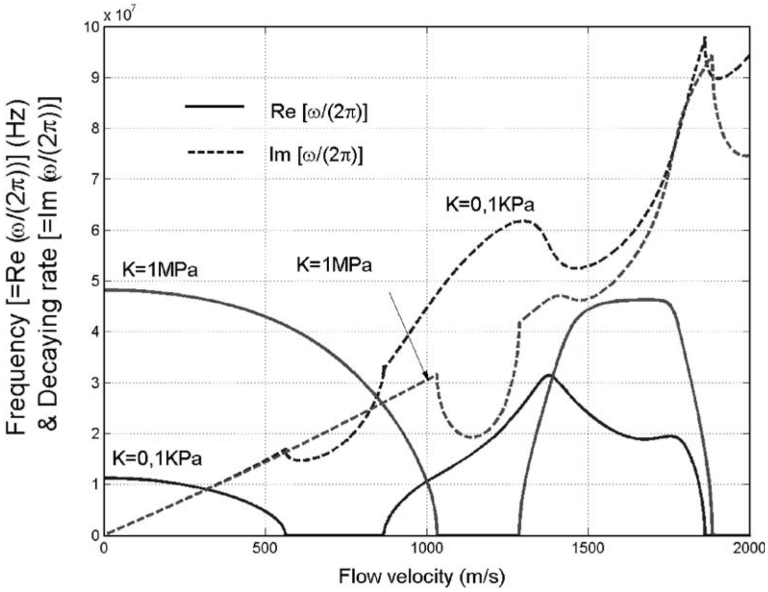

Fig. 11. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case II, $L/2R = 50$, mode 1).

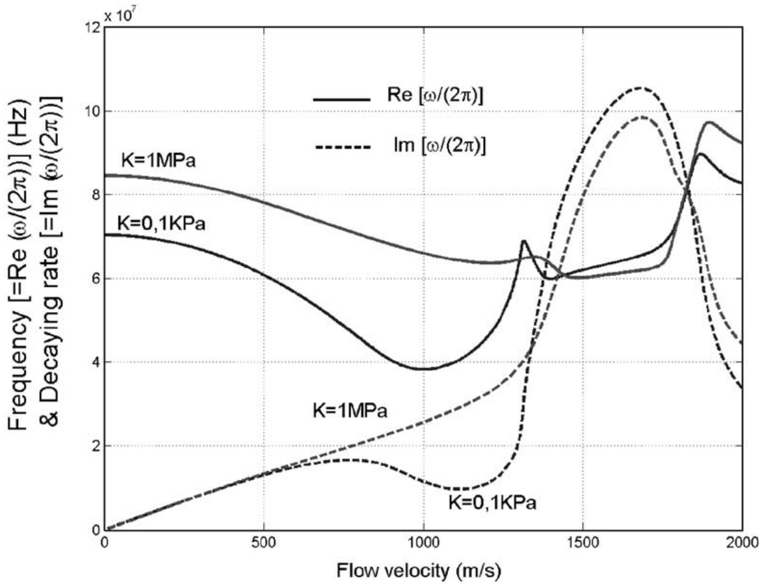

Fig. 12. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case II, $L/2R = 50$, mode 2).

(4) Without a surrounding elastic medium ($K = 0$), the critical flow velocity for flutter instability, at which the decaying rate of amplitude changes from positive to negative and thus the amplitude starts to grow, is inversely proportional to the aspect ratio ($L/2R$), while both vibrational frequency ($=\mathrm{Re}(\omega/(2\pi))$) and the decaying rate ($=\mathrm{Im}(\omega/(2\pi))$) are inversely proportional to square of the aspect ratio

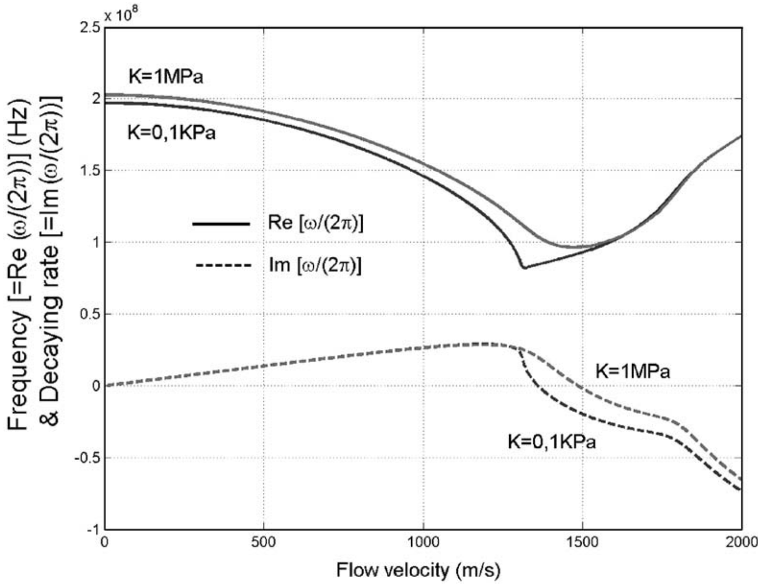

Fig. 13. Frequency and the decaying rate of amplitude as a function of the flow velocity (Case II, $L/2R = 50$, mode 3).

($L/2R$) (Figs. 2–13). For example, in Case I, the critical flow velocities U_c for flutter instability (of mode 2) are 7564 m/s and 1512 m/s for $L/2R = 10$ and 50, respectively (Table 1). On the other hand vibrational frequencies ($=\text{Re}(\omega/(2\pi))$) of mode 1 at $U = 0$, at which the decaying rate is zero for both cases, are 380 MHz and 15 MHz for $L/2R = 10$ and 50, respectively. On the other hand, for the same aspect ratio ($L/2R$), the effect of internal flow on vibrational frequencies ($=\text{Re}(\omega/(2\pi))$) and the decaying rate ($=\text{Im}(\omega/(2\pi))$) are more significant for thin CNTs than for thicker CNTs. For example, for $L/2R = 10$, flutter instability occurs at 7564 m/s for thicker CNT of smaller innermost radius (Case I), and at 6803 m/s for thin CNT of larger innermost radius (Case II) (Table 1). This is attributed to the fact that when the outermost radius is not very different, the restoring flexural force of CNTs of smaller innermost radius is significantly larger than that of CNTs of larger innermost radius, and thus the destabilizing centrifugal force overcomes the restoring flexural force for CNTs of smaller innermost radius only at a higher flow velocity.

(5) As studied in our recent work (Yoon et al., 2005), even a compliant surrounding elastic medium has a significant effect on flow-induced instability of supported CNTs conveying fluid. Here, Let us consider the role of a surrounding elastic medium in flutter instability of cantilever CNTs. First, with a very soft

Table 1
Critical flow velocity for flutter instability of cantilever CNTs

Winkler const. (K)		0		1 KPa		1 MPa		1 GPa
Aspect ratio ($L/2R$)		10	50	10	50	10	50	10
Case I	Mode 1	–	–	–	–	–	–	–
	Mode 2 (m/s)	7560	1510	7560	1510	7560	1790	10080
	Mode 3	–	–	–	–	–	–	–
Case II	Mode 1	–	–	–	–	–	–	–
	Mode 2 (m/s)	12070	2410	12070	2420	12070	2440	–
	Mode 3 (m/s)	6800	1360	6800	1360	6800	1490	7920

surrounding elastic medium, such as bio-tissue with $K = 1$ KPa (Shull, 2002), all phenomena for vibration and flutter instability of cantilever CNTs are similar as those obtained above in the absence of a surrounding elastic medium ($K = 0$). For example, the critical flow velocity U_c of Case II when $L/2R = 50$ and $K = 1$ KPa is 1364 m/s, which is almost same as 1361 m/s of the same case with $K = 0$ (Table 1). Hence, it is concluded that a very soft surrounding elastic medium (say $K \leqslant 1$ KPa) has almost no effect on vibrational frequencies and the decaying rate of cantilever CNTs conveying fluid.

(6) When the Winkler constant of the surrounding elastic medium increases to 1 MPa, the surrounding elastic medium still doesn't make any significant difference for CNTs of smaller aspect ratio $L/2R = 10$. However, if the aspect ratio is larger (such as $L/2R = 50$), a surrounding elastic medium with $K = 1$ MPa significantly reduces the sensitivity of vibrational frequency ($=\mathrm{Re}(\omega/(2\pi))$) and the decaying rate ($=\mathrm{Im}(\omega/(2\pi))$) to the internal flow velocity U. For example, the critical flow velocity U_c of Case I with $L/2R = 50$ increases from 1512 m/s for $K = 0$ to 1789 m/s for $K = 1$ MPa (Table 1). Hence, the role of a surrounding elastic medium with $K = 1$ MPa becomes significant for CNTs of larger aspect ratio, such as those shown in Figs. 5–7, and 11–13.

(7) When a moderately stiffer surrounding elastic medium (such as polymer with $K = 1$ GPa) is considered, the surrounding elastic medium has a more significant effect on vibrational frequency and the decaying rate. For example, even for a short CNT ($L/2R = 10$), the critical flow velocity U_c of Case I increases from 7564 m/s when $K = 0$ to 10076 m/s when $K = 1$ GPa (Table 1). Therefore, it is concluded that a moderately stiff surrounding elastic medium (with $K = 1$ GPa) has a significant effect on vibrational frequencies and flutter instability even for CNTs of small aspect ratio $L/2R = 10$.

4. Conclusions

The influence of internal moving fluid on free vibration and flow-induced flutter instability of cantilever CNTs is studied in this paper. Unlike supported CNTs which lose stability by static buckling, cantilever CNTs lose stability by flutter at a certain critical flow velocity. Our results indicate that internal moving fluid has a substantial effect on vibrational frequencies and the decaying rate of amplitude, and the critical flow velocity for flutter instability in some cases may fall within the range of practical significance. On the other hand, our results indicate that a moderately stiff surrounding elastic medium can significantly reduce the effect of internal moving fluid on vibrational frequencies and suppress or eliminate the flow-induced flutter instability, while a very soft surrounding elastic medium has almost no effect on vibrational frequencies and the decaying rate of amplitude. We believe that these results provide useful data for the proposed application of CNTs as nanopipes conveying fluid. Also, these data may be used to develop a possible method to estimate the internal flow velocity by detecting the changes in resonant frequencies and the decaying rate of amplitude of CNTs conveying fluid.

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